MIMO Discrete Nonlinear Adaptive NN Control using a Learning Algorithm Based on Kalman Filtering

Edgar N. Sanchez and Alma Y. Alanis

CINVESTAV, Unidad Guadalajara, Apartado Postal 31-438,
Plaza La Luna, Guadalajara, Jalisco, C.P. 45091, Mexico
sanchez@gdl.cinvestav.mx

Abstract. This paper deals with the adaptive tracking problem for MIMO nonlinear systems in discrete-time in presence of bounded disturbances. In this paper, a high order neural network structure is used to emulate the control law designed by the backstepping method. The paper includes the respective stability analysis on the basis of the Lyapunov approach for the extended Kalman filter (EKF)-based NN training algorithm.
Keywords. High order neural networks, extended Kalman filter (EKF), neural network learning, adaptive control, backstepping.

1 Introduction

Neural networks (NN) have become a well-established methodology as exemplified by their applications to identification and control of general nonlinear and complex systems. In particular, the use of high order neural networks for modeling and learning has increased recently [7]. There are some publications about the trajectory tracking using neural networks ([7], [5], [6]); in most of them the methodology is based on a Lyapunov method. However most of those works were developed for continuous-time systems. For discrete-time systems, the control problem is more complex due to the couplings among subsystems, inputs and outputs. Besides the difficulty of couplings, the noncausal problem is another difficulty that has to be solved when constructing adaptive controllers for discrete-time systems in strict feedback form [2]. Few results have been published in comparison with those for continuous-time domain ([2], [3]). By other hand discrete-time neural networks are more convenient for real-time applications.

The method presented here has some advantages; the first one is the application to MIMO nonlinear systems in discrete-time; the second one is to guarantee the boundeness of the error in presence of disturbances, and the third one is the use of a transformation to avoid the causality problem. Finally, this paper also proposes the use of High Order Neural Networks (HONN) to emulate the control law designed by the backstepping-method [3]. In this paper, we propose a modification of the method presented in [3]. This modification basically consist in changing the neural network learning rule propose there by a learning algorithm based on the (EKF) [1].

© *A. Gelbukh, C. Yáñez Márquez, O. Camacho Nieto (Eds.)*
Advances in Artificial Intelligence and Computer Science
Research on Computing Science 14, 2005, pp. 13-24

2 Mathematical preliminaries

Through this paper we use k as the step sampling $k \in 0 \cup \square^{+}$, $|\bullet|$ for the absolute value, $\|\bullet\|$ for the Euclidian norm for vectors and for any adequate norm for matrices. For more details related to this section see [3].

Consider a MIMO nonlinear system:

$$x(k+1) = F(x(k), u(k)) \tag{1}$$

Definition 1. The solution of (1) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of $\Re^{\sum_{i=1}^{n} n_i}$ and all $x(k_0) \in \Omega$, there exists an ε>0 and a number $N(\varepsilon, x(k_0))$ such that $\|x(k)\|<\varepsilon$ for all $k \geq k_0 + N$. In other words, the solution of (1) is said to be SGUUB if, for any apriori given (arbitrarily large) bounded set Ω and any apriori given (arbitrarily small) set Ω_0, which contains (0,0) as an interior point, there exists a control u, such that every trajectory of the closed loop system starting from Ω enters the set $\Omega_0 = \{x(k) \mid \|x(k)\| < \varepsilon\}$, in a finite time and remains in it thereafter [3].

Definition 2. Let $V(x(k))$ be a Lyapunov function of a discrete-time system, which satisfies the following properties [3]:

$$\begin{aligned} \gamma_1(\|x(k)\|) &\leq V(x(k)) \leq \gamma_2(\|x(k)\|) \\ V(x(k+1)) - V(x(k)) &= \Delta V(x(k)) \leq -\gamma_3(\|x(k)\|) + \gamma_3(\zeta) \end{aligned}$$

where ζ is a positive constant, $\gamma_1(\bullet)$ and $\gamma_2(\bullet)$ are strictly increasing functions, and $\gamma_3(\bullet)$ is a continuous, nondecreasing function. Thus if

$$\Delta V(x) < 0 \;\; for \;\; \|x(k)\| > \zeta$$

then $x(k)$ is uniformly ultimately bounded, i.e. there exists a time instant k_T, such $\|x(k)\| < \zeta, \forall\ k < k_T$.

Lemma 1: Consider the linear time varying discrete-time system given by

$$\begin{aligned} x(k+1) &= A(k)x(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

where $A(k)$, B and C are appropriately dimensional matrices. Let $\Phi(k(1), k(0))$ be the state-transition matrix corresponding to $A(k)$ for system (2), i.e.

$$\Phi(k(1), k(0)) = \prod_{k=k(0)}^{k=k_1-1} A(k)$$

If $\|\Phi(k(1),k(0))\|<1\ \forall k(1)>k(0)>0$, then the system (2) is 1) globally exponentially stable for the unforced system and 2) Bounded Input-Bounded Output (BIBO) stable.

3 Discrete-time High Order Neural Networks

Under certain conditions, it has been proven that several approximation methods, such NN, have function approximation abilities, and have been frequently used as function approximators. There are several types of NN that have been frequently used. For clarity and simplicity in this paper the HONN is consider:

$$\begin{aligned} \phi(w,z) &= w^T S(z), \quad w\in \Re^{l\times p}, S(z)\in \Re^{l} \\ S(z) &= [s_1(z), s_2(z), \cdots, s_l(z)]^T \\ s_i(z) &= \prod_{j\in I_i} [s(z_j)]^{d_j(i)}, \quad i=1,2,\cdots,l \end{aligned} \tag{3}$$

where $z=[z_1,z_2,\cdots,z_q]^T \in \Omega_z \subset \Re^q$ are positive integer; l denotes the NN node number; p is the dimension of the function vector; $\{I_1, I_2, \cdots, I_l\}$ is a collection of not-ordered subsets of $\{1,2,\cdots,q\}$ and $d_j(i)$ are nonnegative integers; w is an adjustable synaptic weight vector, $s(z_j)$ is chosen as the hyperbolic tangent function:

$$S(x)=\frac{e^{z_j}-e^{-z_j}}{e^{z_j}+e^{-z_j}} \tag{4}$$

For a desired function $u^*(z)$, there exists ideal weights w^* such that smooth function u^* can be approximated by an ideal NN on a compact set $\Omega_z \subset \Re^q$

$$u^* = w^{*T} S(z) + \varepsilon_z$$

where ε_z is the bounded NN approximation error, which can be reduced by increasing the number of the adjustable weights. The ideal weight matrix w^* is an artificial quantity required for analytical purpose [3], [6]. In general it is assumed that there exist unknown but constant weights w^*, whose estimate is $w(k)$. Hence it is possible to define:

$$\tilde{w}(k) = w(k) - w^*$$

4 The EKF Training Algorithm

Kalman filtering (KF) estimates the state of a linear system with additive state and output white noises [1], [4]. For KF-based neural network training, the network weights become the states to be estimated, with the error between the neural network output and the desired output being considered; this error is considered as additive white noise. For identification, the desired output is information generated by the plant. Due to the fact that the neural network mapping is nonlinear, an extended Kalman Filtering (EKF)-type is required. The training goal is to find the optimal weight values that minimize the prediction errors (the differences between the measured outputs and the neural network outputs). The EKF-based training algorithm is described by:

$$K(k) = P(k)H(k)[R(k) + H^T(k)P(k)H(k)]^{-1} \tag{6}$$

$$w(k+1) = w(k) + K(k)[y(k) - \hat{y}(k)] \tag{7}$$

$$P(k+1) = P(k) - K(k)H^T(k)P(k) + Q(k) \tag{8}$$

where $P(k) \in \Re^{l \times l}$ is the prediction error covariance matrix at step k, $w \in \Re^{l}$ is the weight (state) vector, l is the respective number of neural network weights, $y \in \Re$ is the measured output, $\hat{y} \in \Re$ is the neural network output, $K \in \Re^{l}$ is the Kalman gain vector, $Q \in R^{l \times l}$ is the NN weight estimation noise covariance matrix, $R \in \Re$, is the error noise covariance, $H \in \Re^{l}$ is a vector, in which each entry (H_{ij}) is the derivative of one of the neural network output, ($\hat{y}_i$), with respect to one neural network weight, (w_j), as follows:

$$H_{ij}(k) = \left. \frac{\partial \hat{y}_i(k)}{\partial w_j(k)} \right|_{w(k)=w(k+1)} \tag{9}$$

where $j = 1, \ldots, l$. Usually P and Q are initialized as diagonal matrices, with entries $P(0)$ and $Q(0)$, respectively. It is important to remark that $H(k)$, $K(k)$ and $P(k)$ for the EKF are bounded; for a detailed explanation of this fact see [8].

5 Controller design

This section closely follows [3], but replacing the σ-modification learning by an EKF one. Consider the following n inputs n outputs discrete-time MIMO nonlinear system with triangular form as shown in (10), where with

$X(k)=\left[x_1^T(k),x_2^T(k),\cdots,x_n^T(k)\right]^T$, $\quad x_j(k)=\left[x_{j,1}(k),x_{j,2}(k),\cdots,x_{j,n_j}(k)\right]$ $u(k)=\left[u_1(k),\cdots,u_n(k)\right]^T\in\Re^n$ and $y(k)=\left[y_1(k),\cdots,y_n(k)\right]^T\in\Re^n$ are the state variables, the system inputs and outputs respectively; $\bar{u}_{j-1}(k)=\left[u_1(k),\cdots,u_{j-1}(k)\right]^T$ $(j=2,\cdots,n)$; $d(k)=\left[d_1(k),\cdots,d_n(k)\right]^T$ is the bounded disturbance vector $\bar{x}_{j-i_j}(k)=\left[x_{j,1}(k),\cdots,x_{j,i_j}(k)\right]^T\in\Re^{i_j}$ denote the first i_j states of the *j*th subsystem; $f_{j,i_j}(\bullet)$ and $g_{j,i_j}(\bullet)$ are smooth nonlinear functions; and j, i_j and n_j are positive integers. Based on [3] it can be seen that each subsystem of (10) is in strict feedback form, which allows the use of the backstepping design technique. Furthermore, noting that the control inputs of the whole system are in triangular form, we may then use backstepping in a nested manner to design stable controllers for this class of system [3].

$$\Sigma_1=\begin{cases} x_{1,i_1}(k+1)=f_{1,i_1}\left(\bar{x}_{i,i_1}(k)\right)+g_{1,i_1}\left(\bar{x}_{i,i_1}(k)x_{i,i_1+1}(k)\right) \\ \vdots \\ x_{1,n_1}(k+1)=f_{1,n_1}\left(X(k)\right)+g_{1,n_1}\left(X(k)u_1(k)\right)+d_1(k) \end{cases} \tag{10}$$

$$\vdots$$

$$\Sigma_n=\begin{cases} x_{n,i_n}(k+1)=f_{n,i_n}\left(\bar{x}_{n,i_n}(k)\right)+g_{n,i_n}\left(\bar{x}_{n,i_n}(k)x_{n,i_n+1}(k)\right) \\ \vdots \\ x_{n,n_n}(k+1)=f_{n,n_n}\left(X(k),\bar{u}_{n-1}\right)+g_{n,n_n}\left(X(k)u_n(k)\right)+d_n(k) \end{cases}$$

By the transformation used in [3] the *j*th subsystem of the original system (10) is equivalent to (11) which is in a sequential decrease cascade form (SDCF) [3], as follows:

$$\begin{aligned} &x_{j,1}(k+n_j)=F_{j,1}\left(\bar{x}_{j,n_j}(k)\right)+G_{j,1}\left(\bar{x}_{j,n_j}(k)\right)x_{j,2}(k+n_j-1) \\ &\vdots \\ &x_{j,n_j-1}(k+2)=F_{j,n_j-1}\left(\bar{x}_{j,n_j}(k)\right)+G_{j,n_j-1}\left(\bar{x}_{j,n_j}(k)\right)x_{j,n_j}(k+1) \\ &x_{j,n_j}(k+1)=F_{j,n_j}\left(X,u_{j-1}(k)\right)+g_{j,n_j}(X)u_j(k)+d_j(k) \\ &y_j(k)=x_{j,1}(k) \end{aligned} \tag{11}$$

For convenience of analysis, define $1\le j\le n$ and $1\le i_j\le n_j-1$.

$$F_{j,i_j}\,\square\,F_{j,i_j}\left(\bar{x}_{j,n_j}(k)\right) \qquad G_{j,i_j}\,\square\,G_{j,i_j}\left(\bar{x}_{j,n_j}(k)\right)$$

$$f_{j,i_j}\,\square\,F_{j,i_j}\left(X(k),\bar{u}_{j-1}(k)\right) \qquad G_{j,i_j}\,\square\,g_{j,i_j}\left(X(k)\right)$$

Then system (11) can be written as

$$
\begin{aligned}
&x_{j,1}\left(k+n_j\right)=F_{j,1}(k)+G_{j,1}(k)\,x_{j,2}\left(k+n_j-1\right)\\
&\vdots\\
&x_{j,n_j-1}(k+2)=F_{j,n_j-1}(k)+G_{j,n_j-1}(k)\,x_{j,n_j}(k)\\
&x_{j,n_j}(k+2)=f_{j,n_j}(k)+g_{j,n_j}(k)\,u_j(k)+d_j(k)\\
&y_j(k)=x_{j,1}(k)
\end{aligned}
\tag{12}
$$

Now, we can define the desired virtual controls and the ideal practical controls for each subsystem, as follows:

$$
\begin{aligned}
&\alpha^{*}_{j,2}(k)\triangleq\varphi_{j,1}\left(\bar{x}_{j,n_j}(k),y_{d_j}\left(k+n_j\right)\right)\\
&\vdots\\
&\alpha^{*}_{j,n_j}(k)\triangleq\varphi_{j,n_j-1}\left(\bar{x}_{j,n_j-1}(k),\alpha^{*}_{j,n_j-1}(k)\right)\\
&u^{*}_j(k)\triangleq\varphi_{j,n_j}\left(X(k),u_{j-1}(k),\alpha^{*}_{j,n_j}(k)\right)\\
&y_j(k)=x_{j,1}(k)
\end{aligned}
\tag{13}
$$

where φ_{j,n_j} $\left(1\le j\le n \text{ and } 1\le i_j\le n_j\right)$ are nonlinear functions. It is obvious that the desired virtual controls $\alpha^{*}_j(k)$ and the ideal control $u^{*}_j(k)$ will drive the output of the jth subsystem to track the desired signal only if the exact system model is known and without disturbances. However in practical applications these two conditions cannot be satisfied. In the following, neural networks will be used to emulate the desired virtual controls, as well the desired practical controls when the conditions establish above are not satisfied. Therefore, in [3] they construct the controls via embedded backstepping without causality contradiction. Let select the virtual controls and practical controls as follows $1\le j\le n$:

$$
\begin{aligned}
&\alpha_{j,i_j}=w_{j,i_j-1}S_{j,i_j-1}\left(z_{j,i_j-1}(k)\right)\\
&u_j=w_{j,n_j}S_{j,n_j}\left(z_{j,n_j}(k)\right)
\end{aligned}
\tag{14}
$$

with

$$
\begin{aligned}
&z_{j,1}(k)=\left[\bar{x}^{T}_{j,n_j}(k),y_{d_j}\left(k+n_j\right)\right]^{T}\\
&z_{j,i_j}(k)=\left[\bar{x}^{T}_{j,n_j}(k),\alpha_{j,i_j}(k)\right]^{T}\\
&z_{j,1}(k)=\left[X(k),\alpha_{j,n_j}(k)\right]^{T}\qquad i_j=2,\cdots,n_j-1
\end{aligned}
$$

where w_{j,i_j} is the estimation of ideal constant w^{*}_{j,i_j} $(1\le j\le n,\ 1\le i_j\le n_j)$. Through the paper the following definition is used:

$$\tilde{w}_{j,i_j}(k) = w_{j,i_j}(k) - w^{*}_{j,i_j} \tag{15}$$

The corresponding weights updating laws are chosen as:

$$w_{j,i_j}(k+1) = w_{j,i_j}(k) - \eta_{j,i_j} K_{j,i_j} e_{j,i_j}(k) \tag{16}$$

with

$$\begin{aligned} &K_{j,i_j}(k) = P_{j,i_j}(k) H_{j,i_j}(k) M^{-1}_{j,i_j}(k) \\ &M_{j,i_j}(k) = \left[R_{j,i_j}(k) H^{T}_{j,i_j}(k) P_{j,i_j}(k) H_{j,i_j}(k)\right]^{-1} \\ &P_{j,i_j}(k+1) = P_{j,i_j}(k) - K_{j,i_j}(k) H^{T}_{j,i_j}(k) P_{j,i_j}(k) + Q_{j,i_j}(k) \\ &H_{j,i_j}(k) = \left[\frac{\partial \hat{y}_{j,i_j}(k)}{\partial w_{j,i_j}(k)}\right] \end{aligned} \tag{17}$$

and $e_{j,i_j}(k)$ denotes the error of each step defined as follows:

$$\begin{aligned} e_{j,1}(k) &= y_{d_1}(k) - y_{j,1}(k) \\ e_{j,2}(k) &= x_{j,2}(k) - \alpha_{j,2}(k) \\ &\vdots \\ e_{j,n_j}(k) &= x_{j,n_j}(k) - u_j(k) \end{aligned} \tag{18}$$

Let consider (3), (5), (17) and (18) then we propose the following theorem.

Theorem 1: For the closed-loop nonlinear MIMO system (10) consisting of control (14) an adaptive law (16), there exists a SGUUB equilibrium, provided that the design parameters are properly chosen. This guarantees that all signals, including the states $X(k)$, the input $u(k)$ and NN weight estimates $w_{j,i_j}(j = 1,\cdots,n;\ i_j = 1,\cdots,n_j)$ are all bounded.

Proof: For the first $n_j - 1$ equations of the jth subsystem $(j = 1,\cdots,n;\ i_j = 1,\cdots,n_j - 1)$ with the fictitious control $\alpha^{*}_{j,i_j}(k)$ approximated by the HONN $\alpha_{j,i_j}(k) = w_{j,i_j-1} S_{j,i_j-1}(z_{j,i_j-1}(k))$ and $e_{j,i_j}(k) = x_{j,i_j}(k) - \alpha_{j,i_j}(k)$. Consider the Lyapunov function candidate

$$V_{j,i_j}(k) = \frac{1}{2} e^{2}_{j,i_j}(k) + \tilde{w}^{T}_{j,i_j}(k) \tilde{w}_{j,i_j}(k) \tag{19}$$

whose first difference is:

$$\begin{aligned} &\Delta V_{j,i_j}(k) = V_{j,i_j}(k+1) - V_{j,i_j}(k) \\ &\Delta V_{j,i_j}(k) = \frac{1}{2} e^{2}_{j,i_j}(k+1) + \tilde{w}^{T}_{j,i_j}(k+1) \tilde{w}_{j,i_j}(k+1) - \frac{1}{2} e^{2}_{j,i_j}(k) - \tilde{w}^{T}_{j,i_j}(k) \tilde{w}_{j,i_j}(k) \end{aligned} \tag{20}$$

From (7) and (6), then

$$\tilde{w}_{j,i_j}(k+1)=\tilde{w}_{j,i_j}(k)+\eta_{j,i_j}K_{j,i_j}(k)e_{j,i_j}(k) \tag{21}$$

Let us define $N_{j,i_j}(k)\in\Re$ by

$$\begin{aligned}N_{j,i_j}(k)=&\,\tilde{w}^T_{j,i_j}(k)\tilde{w}_{j,i_j}(k)+2\eta_{j,i_j}e_{j,i_j}(k)\tilde{w}_{j,i_j}(k)K_{j,i_j}(k)\\&+\eta^2_{j,i_j}e^2_{j,i_j}(k)K^T_{j,i_j}(k)K_{j,i_j}(k)\end{aligned} \tag{22}$$

From (18), then

$$\begin{aligned}&e_{j,i_j}(k+1)=e_{j,i_j}(k)+\Delta e_{j,i_j}(k)\\&e^2_{j,i_j}(k+1)=e^2_{j,i_j}(k)+2e_{j,i_j}(k)\Delta e_{j,i_j}(k)+\left(\Delta e_{j,i_j}(k)\right)^2\end{aligned} \tag{23}$$

where $\Delta e_{j,i_j}(k)$ is the error difference. Using (22) and (23) in (20):

$$\begin{aligned}\Delta V_{j,i_j}(k)=&\,e_{j,i_j}(k)\Delta e_{j,i_j}(k)+\frac{1}{2}\left(\Delta e_{j,i_j}(k)\right)^2+2\eta_{j,i_j}e_{j,i_j}(k)K_{j,i_j}(k)\\&+\eta^2_{j,i_j}e^2_{j,i_j}(k)K^T_{j,i_j}(k)K_{j,i_j}(k)\end{aligned} \tag{24}$$

From (14), we obtain

$$\frac{\partial e_{j,i_j}(k)}{\partial w_{j,i_j}(k)}=-\frac{\partial \alpha_{j,i_j}(k)}{\partial w_{j,i_j}(k)} \tag{25}$$

Let as approximating (25) and substituting (17) and (5) yields

$$\Delta e_{j,i_j}(k)=-\eta_{j,i_j}H^T_{j,i_j}(k)K_{j,i_j}(k)e_{j,i_j}(k)$$

Defining

$$\gamma_{j,i_j}=\min\left\|H^T_{j,i_j}(k)P_{j,i_j}(k)H_{j,i_j}(k)M_{j,i_j}(k)\right\| \tag{26}$$

with $M_{j,i_j}(k)$ as in (17), (26) can be rewritten as

$$\Delta e_{j,i_j}(k)\le-\eta_{j,i_j}\gamma_{j,i_j}e_{j,i_j}(k) \tag{27}$$

Using (27) in (24), then

$$\begin{aligned}\Delta V_{j,i_j}(k)\le&-\eta_{j,i_j}\gamma_{j,i_j}e^2_{j,i_j}(k)+\frac{1}{2}\eta^2_{j,i_j}\gamma^2_{j,i_j}e^2_{j,i_j}(k)\\&+2\eta_{j,i_j}e_{j,i_j}(k)\tilde{w}^T_{j,i_j}(k)K_{j,i_j}(k)\\&+\eta^2_{j,i_j}e^2_{j,i_j}(k)K^T_{j,i_j}(k)K_{j,i_j}(k)\end{aligned}$$

$$\Delta V_{j.i_j}(k) \le -\eta_{j.i_j}\gamma_{j.i_j}\left|e_{j.i_j}(k)\right|^2 + \frac{1}{2}\eta_{j.i_j}^2\gamma_{j.i_j}^2\left|e_{j.i_j}(k)\right|^2 \\ +2\eta_{j.i_j}\left|e_{j.i_j}(k)\right|\left\|\tilde{w}_{j.i_j}^T(k)\right\|\left\|K_{j.i_j}(k)\right\| + \eta_{j.i_j}^2\left|e_{j.i_j}(k)\right|^2\left\|K_{j.i_j}(k)\right\|^2 \tag{28}$$

The weight adaptation dynamics (21) can be written as

$$\tilde{w}_{j.i_j}^T(k+1) = \tilde{w}_{j.i_j}^T(k) + \eta_{j.i_j}K_{j.i_j}^T(k)e_{j.i_j}(k) = A_{j.i_j}(k)\tilde{w}_{j.i_j}^T(k) + \eta_{j.i_j}v_{z_{j.i_j}}(k)$$

with

$$A_{j.i_j}(k) = \left[I - \eta_{j.i_j}K_{j.i_j}^T(k)S\left(z_{j.i_j}(k)\right)\right] \text{ and } v_{z_{j.i_j}}(k) = K_{j.i_j}^T(k)\varepsilon_{j.i_j} \tag{29}$$

As in [9], in this paper the plant (10) is assumed to be BIBO, $\varepsilon_{z_{j.i_j}}$ and $S\left(z_{j.i_j}(k)\right)$ are bounded. Hence $A_{z_{j.i_j}}$ satisfies Lemma 1 subsequently $\tilde{w}_{j.i_j}(k)$ is bounded. Then in (28) $\Delta V_{j.i_j}(k) \le 0$, once $\left|e_{j.i_j}(k)\right| > \kappa_{j.i_j}$ with $\kappa_{j.i_j} = \dfrac{4\bar{w}_{j.i_j}\bar{K}_{j.i_j}}{2\gamma_{j.i_j} - \eta_{j.i_j}\gamma_{j.i_j}^2 - 2\eta_{j.i_j}\bar{K}_{j.i_j}^2}$, where $\bar{w}_{j.i_j}$ is the upper bound of $\tilde{w}_{j.i_j}(k)$ and $\bar{K}_{j.i_j}$ is the upper bound of $K_{j.i_j}(k)$ [8]; furthermore $\gamma_{j.i_j}>0$ and $\eta_{j.i_j} > 0$. This implies the boundness of $V_{j.i_j}(k)$ for $k \ge 0$ which leads to the SGUUB of $e_{j.i_j}(k)$.

For the first $n_j - 1$ equations of (10), we have show that their stability can be guaranteed by suitable chosen the virtual control design parameters. Let us consider the last equation of the jth subsystem $(j = 1, \cdots, n)$ with the fictitious control $u_j^*(k)$ approximated by the HONN $u_j(k) = w_{j.n_j}S_{j.n_j}\left(z_{j.n_j}(k)\right)$ and $e_{j.n_j}(k) = x_{j.n_j}(k) - u_j(k)$. Consider the Lyapunov function candidate

$$V_{j.n_j}(k) = \frac{1}{2}e_{j.n_j}^2(k) + \tilde{w}_{j.n_j}^T(k)\tilde{w}_{j.n_j}(k) \tag{30}$$

whose first difference is:

$$\Delta V_{j.n_j}(k) = \frac{1}{2}e_{j.n_j}^2(k+1) + \tilde{w}_{j.n_j}^T(k+1)\tilde{w}_{j.n_j}(k+1) - \frac{1}{2}e_{j.n_j}^2(k) - \tilde{w}_{j.n_j}^T(k)\tilde{w}_{j.n_j}(k) \tag{31}$$

In a similar way as the procedure above, (31) can be written as

$$\Delta V_{j.n_j}(k) = e_{j.n_j}(k)\Delta e_{j.n_j}(k) + \frac{1}{2}\left(\Delta e_{j.n_j}(k)\right)^2 + 2\eta_{j.n_j}e_{j.n_j}(k)K_{j.n_j}(k) \\ + \eta_{j.n_j}^2 e_{j.n_j}^2(k)K_{j.n_j}^T(k)K_{j.n_j}(k) \tag{32}$$

with

$$\Delta e_{j,n_j}(k) = -\eta_{j,n_j} H_{j,n_j}^T(k) K_{j,n_j}(k) e_{j,n_j}(k) \tag{33}$$

Defining

$$\gamma_{j,n_j} = \min \left\| H_{j,n_j}^T(k) P_{j,n_j}(k) H_{j,n_j}(k) M_{j,n_j}(k) \right\|$$

with $M_{j,n_j}(k)$ as in (17), (33) can be rewritten as

$$\Delta e_{j,n_j}(k) \leq -\eta_{j,n_j} \gamma_{j,n_j} e_{j,n_j}(k) \tag{34}$$

Using (34) in (32), then

$$\begin{aligned}\Delta V_{j,n_j}(k) \leq & -\eta_{j,n_j} \gamma_{j,n_j} \left| e_{j,n_j}(k) \right|^2 + \frac{1}{2} \eta_{j,n_j}^2 \gamma_{j,n_j}^2 \left| e_{j,n_j}(k) \right|^2 \\ & + 2\eta_{j,n_j} \left| e_{j,n_j}(k) \right| \left\| \tilde{w}_{j,n_j}^T(k) \right\| \left\| K_{j,n_j}(k) \right\| + \eta_{j,n_j}^2 \left| e_{j,n_j}(k) \right|^2 \left\| K_{j,n_j}(k) \right\|^2\end{aligned} \tag{35}$$

The weight adaptation dynamics can be written as

$$\tilde{w}_{j,i_j}^T(k+1) = A_{j,i_j}(k) \tilde{w}_{j,i_j}^T(k) + \eta_{j,i_j} v'_{z_{j,i_j}}(k)$$

with

$$A_{j,n_j}(k) = \left[I - \eta_{j,n_j} K_{j,n_j}^T(k) S\left(z_{j,n_j}(k)\right) \right]; v'_{j,n_j}(k) = K_{j,n_j}(k) \left(\varepsilon_{z_{j,i_j}} + d_j(k) \right) \tag{36}$$

As in [9], in this paper the plant (10) is assumed to be BIBO, $\varepsilon_{z_{j,n_j}}$, $d_j(k)$ and $S\left(z_{j,n_j}(k)\right)$ are all bounded. Hence $A_{z_{j,n_j}}$ satisfies Lemma 1, subsequently $\tilde{w}_{j,n_j}(k)$ is bounded too. Then in (35) $\Delta V_{j,n_j}(k) \leq 0$, once $\left| e_{j,n_j}(k) \right| > \kappa_{j,n_j}$ with $\kappa_{j,n_j} = \dfrac{4\bar{w}_{j,n_j} \bar{K}_{j,n_j}}{2\gamma_{j,n_j} - \eta_{j,n_j} \gamma_{j,n_j}^2 - 2\eta_{j,n_j} \bar{K}_{j,n_j}^2}$, where $\bar{w}_{j,n_j}$ is the upper bound of $\tilde{w}_{j,n_j}(k)$ and $\bar{K}_{j,n_j}$ is the upper bound of $K_{j,n_j}(k)$ [8]; furthermore $\gamma_{j,n_j} > 0$ and $\eta_{j,n_j} > 0$. This implies the boundness of $V_{j,n_j}(k)$ for $k \geq 0$ which leads to the SGUUB of $e_{j,n_j}(k)$.

6 Simulation Results

Consider the following MIMO discrete-time nonlinear system:

$$\begin{aligned} x_{1,1}(k+1) &= f_{1,1}\left(\bar{x}_{1,1}(k)\right) + g_{1,1}\left(\bar{x}_{1,1}(k)\right) x_{1,2}(k) \\ x_{1,2}(k+1) &= f_{1,2}\left(\bar{x}_{1,2}(k)\right) + g_{1,2}\left(\bar{x}_{1,2}(k)\right) u_1(k) + d_1(k) \end{aligned}$$

$$x_{2,1}(k+1) = f_{2,1}(\bar{x}_{2,1}(k)) + g_{1,1}(\bar{x}_{2,1}(k))x_{2,2}(k)$$
$$x_{2,2}(k+1) = f_{2,2}(\bar{x}_{2,2}(k)) + g_{2,2}(\bar{x}_{2,2}(k))u_2(k) + d_2(k)$$
$$y_1(k) = x_{1,1}(k) \qquad\qquad y_2(k) = x_{2,1}(k)$$

with

$$f_{1,1}(\bar{x}_{1,1}(k)) = \frac{x_{1,1}^2(k)}{1+x_{1,1}^2(k)}; \quad g_{1,1}(\bar{x}_{1,1}(k)) = 0.3; \quad g_{1,2}(\bar{x}_{1,1}(k)) = 1$$
$$f_{1,2}(\bar{x}_{1,2}(k)) = \frac{x_{1,1}^2(k)}{1+x_{1,2}^2(k)+x_{2,1}^2(k)+x_{2,2}^2(k)}$$
$$d_1(k) = 0.1\cos(0.05k)\cos(x_{1,1}(k))$$
$$f_{2,1}(\bar{x}_{2,1}(k)) = \frac{x_{2,1}^2(k)}{1+x_{2,1}^2(k)}; \quad g_{2,1}(\bar{x}_{2,1}(k)) = 0.2; \quad g_{2,2}(\bar{x}_{1,1}(k)) = 1$$
$$f_{2,2}(\bar{x}_{2,2}(k), u_1(k)) = \frac{x_{2,1}^2(k)u_1^2(k)}{1+x_{1,1}^2(k)+x_{2,1}^2(k)+x_{2,2}^2(k)}$$
$$d_2(k) = 0.1\cos(0.05k)\cos(x_{2,1}(k))$$

The control objective is to drive the output $y(k) = [y_1(k), y_2(k)]^T$ of the system to follow the desired reference signals:

$$y_{d_1} = 0.5 + \frac{1}{4}\cos\left(\frac{\pi Tk}{4}\right) + \sin\left(\frac{\pi Tk}{2}\right) \; ; \; y_{d_2} = 0.5 + \frac{1}{4}\sin\left(\frac{\pi Tk}{4}\right) + \sin\left(\frac{\pi Tk}{2}\right)$$

with $T = 0.01$. The initial conditions for the system are $x_{1,1}(0) = 0$, $x_{1,2}(0) = 0$, and $x_{2,1}(0) = 0$, $x_{2,2}(0) = 0$. All the weights, the virtual and practice controls are initialized as random numbers. The results are presented in Figure 1 as follows. The first two figures portray the tracking performance of the two outputs of the plant and their references, respectively and the third figure displays the weights performance.

Conclusions

This paper has presented an application of HONN to solve the tracking problem for a specific class of MIMO nonlinear systems in discrete-time. The training of the neural network was performed on-line using an extended Kalman filter. The boundness of the tracking error was established on the basis of the Lyapunov approach. The purpose of this paper is to improve the tracking performance of that work by mean of the use of the EKF as the neural network learning algorithm; this approach is validated by the simulation results presented above. The HONN training with the learning algorithm based in EKF present good performance even in presence of larger bounded disturbances.

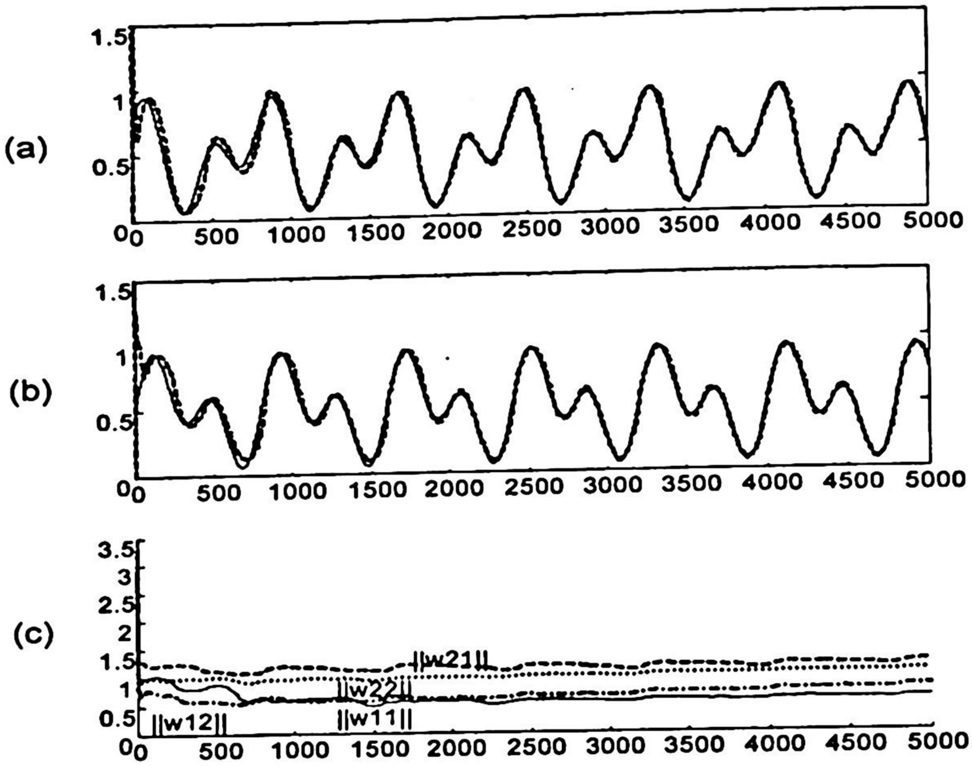

Fig. 1. *a*) Tracking performance $y_1(k)$ (solid line) and $y_{d_1}(k)$ (dashed line); *b*) Tracking performance $y_2(k)$ (solid line) and $y_{d_2}(k)$ (dashed line); *c*) Weights performance

References

1. Chui C. K. and Chen G., Kalman Filtering with Real-Time Applications, Springer-Verlag, New York, 1st ed. 1987, 3rd ed. 1998.
2. Ge S. S., Li G. Y. and Lee T. H., "Adaptive NN control for a class of strict-feedback discrete-time nonlinear systems", Automatica, Vol 39, no. 5, pp 807-819, May, 2003.
3. Ge S. S., Zhang J and Lee T. H., "Adaptive neural network control for a class of MIMO nonlinear systems with disturbances in discrete-time", IEEE Transactions on Systems, Man and Cybernetics, Part B, Vol. 34, no. 4, August, 2004.
4. Grover R. and Hwang P. Y. C., Introduction to Random Signals and Applied Kalman Filtering, 2nd ed., Wiley, New York, 1992.
5. Poznyak A. S., Sanchez E. N. and Yu W., Differential Neural Networks for Robust Nonlinear Control, World Scientific, Singapore, 2001.
6. Rovithakis G. A. and Chistodoulou M. A., Adaptive Control with Recurrent High -Order Neural Networks, Springer Verlag, New York, USA, 2000.
7. Sanchez E. N. and Ricalde L. J., "Trajectory tracking via adaptive recurrent neural control with input saturation," Proceedings of International Joint Conference on Neural Networks'03, Portland, Oregon, USA, July 2003.
8. Song Y. and Grizzle J. W., "The extended Kalman Filter as Local Asymptotic Observer for Discrete-Time Nonlinear Systems", Journal of Mathematical Systems, Estimation and Control, Vol. 5, No. 1, pp. 59-78, Birkhauser-Boston, 1995.
9. Yu W. and Li X., "Nonlinear system identification using discrete-time recurrent neural networks with stable learning algorithms", Information Sciences, Vol. 158, pp. 131-147, 2004.